Fast model of space-variant blurring and its application to deconvolution in astronomy

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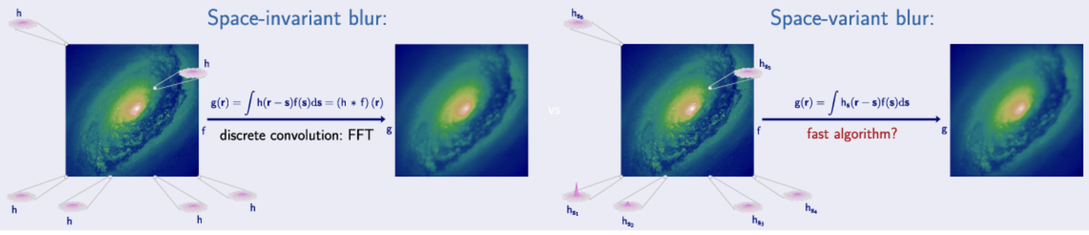
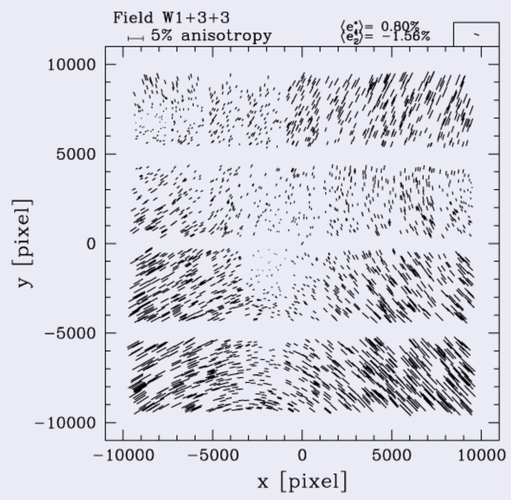
Rationale

In many applications, the point spread function (PSF) varies spatially. This is the case in wide-field imaging in astronomy, due to optical aberrations of the instrument, or imperfect correction of adaptive optics systems away from the reference stars.

PSF are also often shift-variant in microscopy (e.g., in confocal microscopy, the PSF widens with depth).

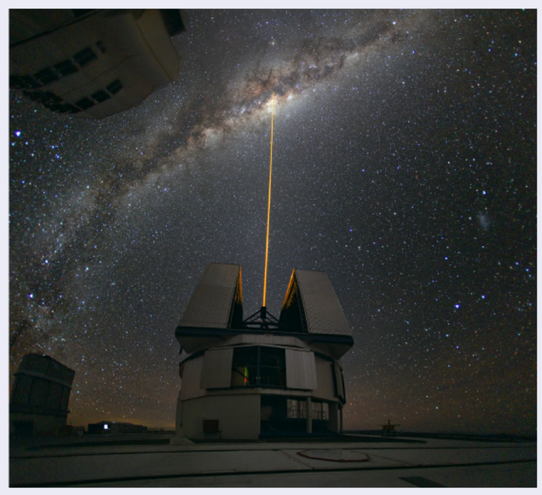
Fast models of space-variant blurring are essential for iterative deblurring.





PSF anisotropy across MegaCam field at the Canada-France Hawaii telescope

source: Hoekstra et al., The Astrophysical Journal, 647 :116, 2006

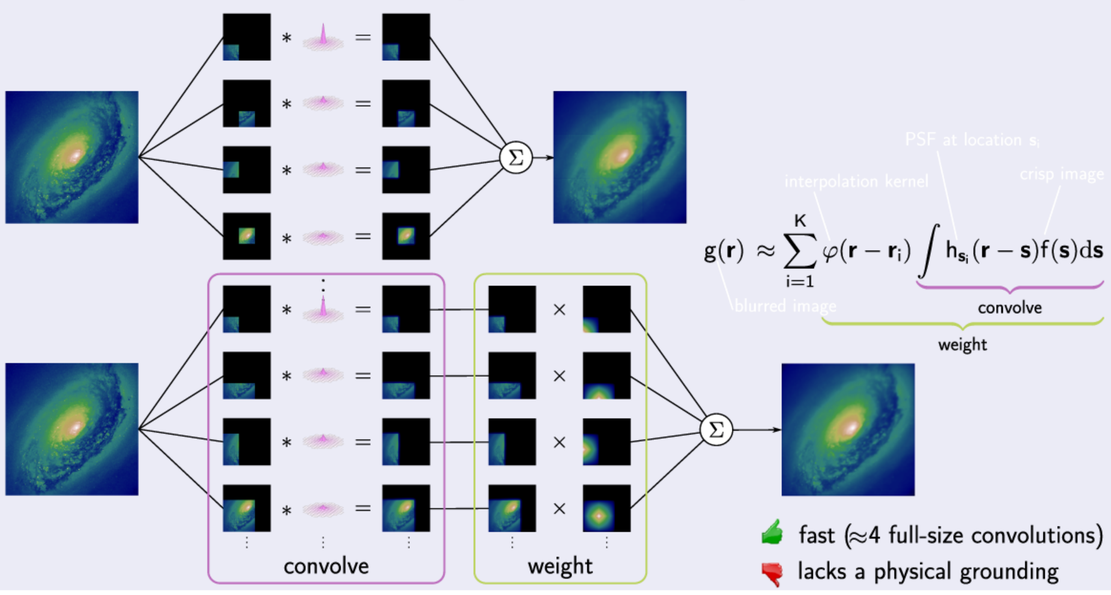


A LASER star used as reference in the adaptive optics system at the VLT (Chile)

source: ESO

Prior works

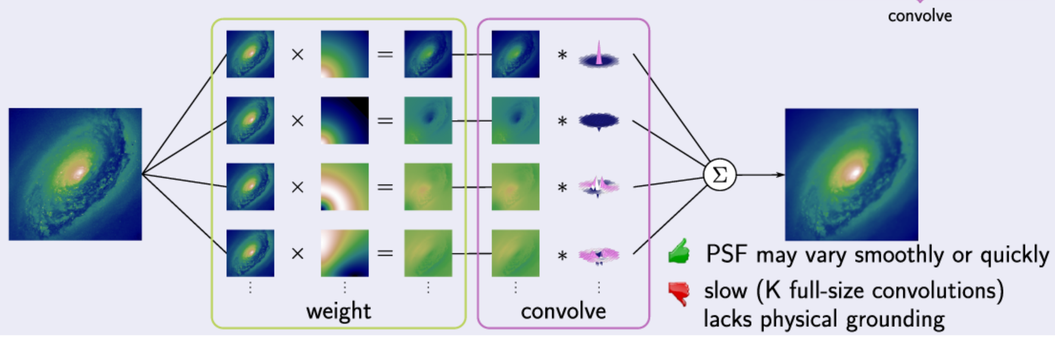
Bloc processing: convolve, then interpolate [Nagy & O'Leary 1998]



PSF decomposition: select the most significant PSF modes by PCA, weight spatially according to local PSF decompositions, then convolve by each mode [Flicker & Rigaut 2005]

$$h_{\mathbf{s}}(\mathbf{r}-\mathbf{s}) \approx \sum_{i=1}^{K} \underbrace{\langle h_{\mathbf{s}} | \tilde{h}_i \rangle}_{w_i(\mathbf{s})} \cdot \tilde{h}_i(\mathbf{r}-\mathbf{s})$$

$$g(\mathbf{r}) = \int h_{\mathbf{s}}(\mathbf{r}-\mathbf{s})f(\mathbf{s})d\mathbf{s} \approx \int \sum_{i=1}^{K} w_i(\mathbf{s}) \cdot \tilde{h}_i(\mathbf{r}-\mathbf{s})f(\mathbf{s})d\mathbf{s} \approx \sum_{i=1}^{K} \int \tilde{h}_i(\mathbf{r}-\mathbf{s})\, w_i(\mathbf{s})f(\mathbf{s})\, d\mathbf{s}$$

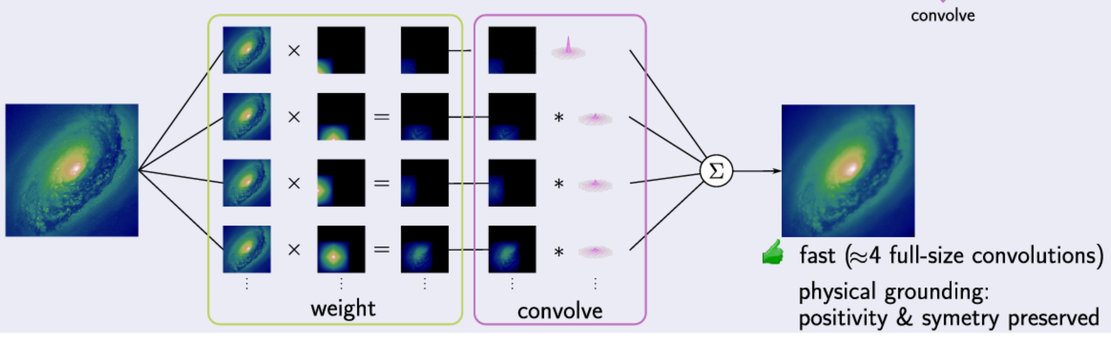


Proposed fast approximations

PSF interpolation: weight with interpolation kernel, then convolve [Hirsch 2010]

$$h_{\mathbf{s}}(\mathbf{r}-\mathbf{s}) \approx \sum_{i=1}^{K} \varphi(\mathbf{s}-\mathbf{s}_i) \cdot h_{\mathbf{s}_i}(\mathbf{r}-\mathbf{s})$$

$$g(\mathbf{r}) = \int h_{\mathbf{s}}(\mathbf{r}-\mathbf{s})f(\mathbf{s})d\mathbf{s} \approx \int \sum_{i=1}^{K} \varphi(\mathbf{s}-\mathbf{s}_i) \cdot h_{\mathbf{s}_i}(\mathbf{r}-\mathbf{s})f(\mathbf{s})d\mathbf{s} \approx \sum_{i=1}^{K} \int h_{\mathbf{s}_i}(\mathbf{r}-\mathbf{s})\, \varphi(\mathbf{s}-\mathbf{s}_i)f(\mathbf{s})\, d\mathbf{s}$$

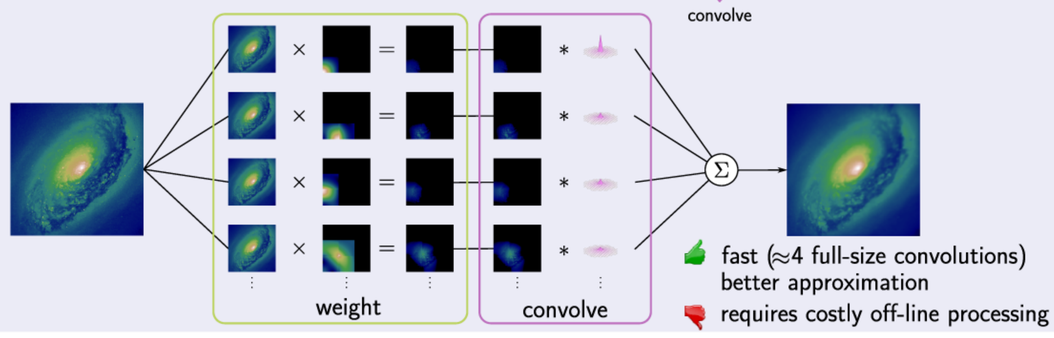


Optimal localized approximation: refine weights and PSF to minimize approximation error

PSF approximation error: $\epsilon^2 = \iint \left[h(\mathbf{r},\mathbf{s}) - \sum_{i=1}^{K} \varphi_i(\mathbf{s}-\mathbf{s}_i) \cdot h_{\mathbf{s}_i}(\mathbf{r}-\mathbf{s}) \right]^2 d\mathbf{r}\, d\mathbf{s}$

Refine weights and convolution kernels to minimize PSF approximation error with localized weights and limited kernel size (alternating minimization).

$$\left(h^\star_{\mathbf{s}_i}, \varphi^\star_i\right)_{i=1..K} = \underset{h_{\mathbf{s}_i}, \varphi_i}{\arg\min}\ \epsilon^2 \qquad g(\mathbf{r}) \approx \sum_{i=1}^{K} \int h^\star_{\mathbf{s}_i}(\mathbf{r}-\mathbf{s})\, \varphi^\star_i(\mathbf{s}-\mathbf{s}_i)f(\mathbf{s})\, d\mathbf{s}$$

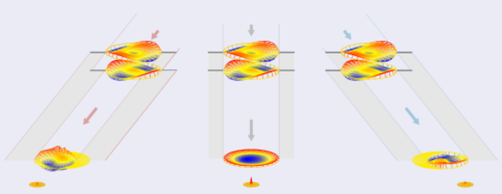


Results

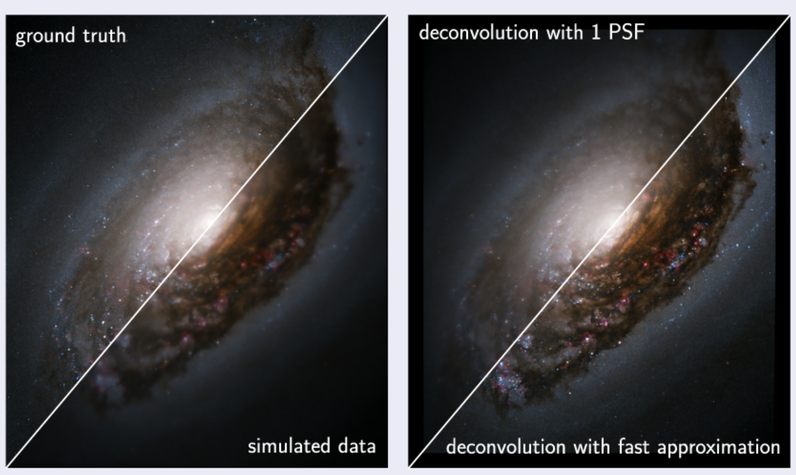
Simulated blur: optical aberrations

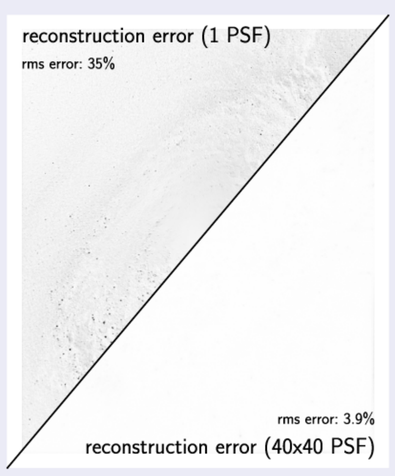
defocus, spherical aberrations, tilt, vignetting

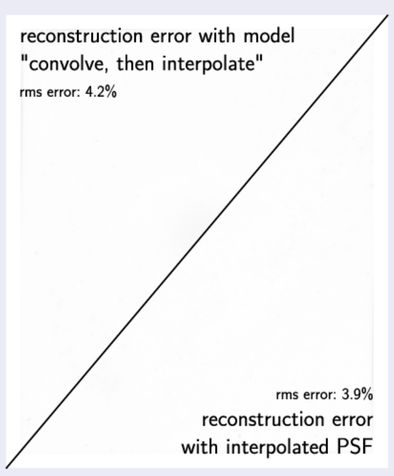
restoration: MAP with smoothed total variation



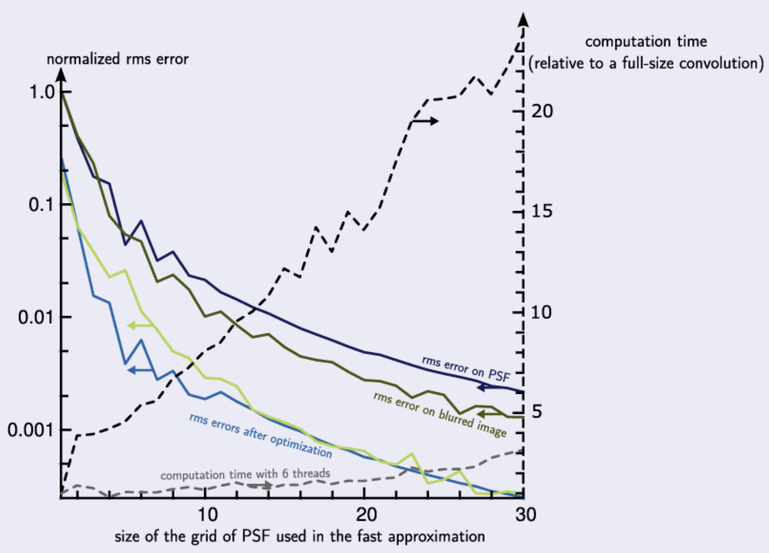
simulated aberrations







Tradeoff between approximation quality and computational cost



[Nagy & O'Leary1998] J.G. Nagy and D.P. O'Leary, "Restoring Images Degraded by Spatially Variant Blur," SIAM J. Sci. Comp., vol. 19, pp. 1063, 1998.
[Flicker & Rigaut 2005] R. Flicker and F. J Rigaut, "Anisoplanatic deconvolution of adaptive optics images.," J Opt Soc Am A, vol. 22, no. 3, pp.504–513, Mar 2005.
[Hirsch et al. 2010] M. Hirsch, S. Sra, B. Scholkopf, and S. Harmeling, "Efficient filter flow for space-variant multiframe blind deconvolution," in IEEE Comp. Vis. Pattern Recogn., 2010, pp. 607–614.

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